The quantum mechanics of photon addition and subtraction

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Photon subtraction and addition do not obey the rules of conventional arithmetic; however, quantum-mechanical arithmetic can be proven experimentally.

In atomic-scale or quantum physics, an electromagnetic field is composed of photons, optical packets so small that a typical laser pointer with $1mW$ of power emits billions of them each second. Being able to take away or add a single photon in a light field at will would be useful for accurate engineering of quantum states. Such pure single photons are ideal for carrying and manipulating information in emerging quantum technologies, but generating them still is very challenging.

In quantum physics, subtraction or addition of a photon is not a deterministic process. Instead, the probability is proportional to the number of photons originally in the electromagnetic field. This is described by the equation $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$, where $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators, respectively, representing the action of subtracting and adding a photon in a field composed of n photons.[1]

Such single-photon operations were limited to pure theoretical discussions until 2004, when Grangier and coworkers found a simple way to subtract a photon[2] from a field. Passing through a beam splitter of high transmittivity, some photons can be split from the initial field. A 'click' detected in the photodetector in the reflected channel means one photon has been subtracted from the field: see Figure 1(a). When the initial field is composed of a small number of photons, the effect is dramatic and may even completely change the characteristics of the field.

For example, let us take an initial single-mode squeezed state generated by 'degenerate' parametric down-conversion, which is an optical nonlinear process that converts a pump photon into two twin photons with half the pump photon energy. (This state is well known for its quantum noise squeezed into one quadrature at the expense of expanded noise in the other quadrature.) In order to visualize the noise properties and characteristics of the squeezed state, a probability-like function, such
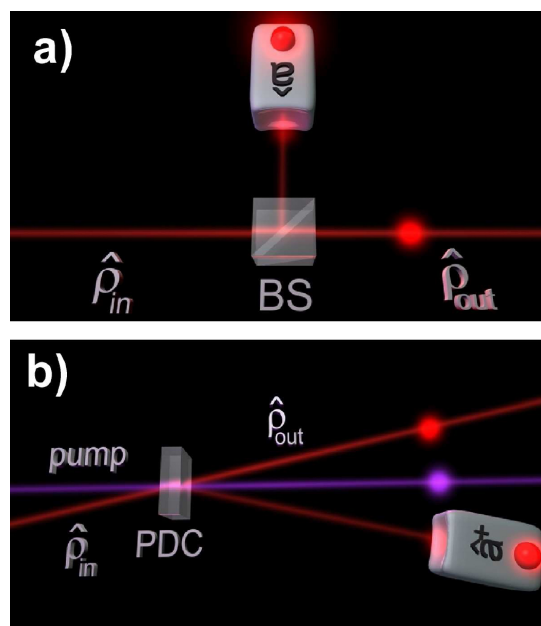

Figure 1. *Setups to subtract (a) and add (b) a single photon from/to a light field. $\hat{\rho}_{in}$ and $\hat{\rho}_{out}$ denote the density operators of the input and output fields; BS is a low-reflectivity beam splitter; PDC is a nonlinear crystal where parametric down-conversion takes place; $\hat{a}$ and $\hat{a}^\dagger$ denote on/off photodetectors.*

as the Wigner function, can be used. When a single photon is subtracted, the Wigner function of the input state dramatically changes from perfect Gaussian to completely non-Gaussian.[3] (The resulting state is very similar to a coherent superposition state that shows the Schrödinger cat paradox, one of the most fundamental interpretations of quantum physics.)

Single-photon addition is experimentally more demanding than subtraction. Recently, we successfully performed a single-photon addition to a coherent field using a 'non-degenerate' parametric down-converter.[4] It differs from the degenerate case in that the two photons are generated into two different spa-

tial modes out of a pump photon. If one photon is detected in one output mode, then we are sure that the other photon is in the other mode, without having to measure it. As shown in Figure 1(b), an initial field input to one mode will gain one extra photon heralded by a photon detected in the conjugate mode. By adding only one photon, any input state is converted into a nonclassical state that cannot be described by classical theory. As seen in Figure 2, we added a photon to a thermal field (the most classical field) which makes the state have negative values in its Wigner function. This is a typical signature of nonclassicality.[5]

In addition, the uncertainty principle and the peculiarity of measurement in quantum physics are closely related to the commutation relation. For photons, the commutation relation takes the form $\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = 1$, which means that the two sequences of photon addition and subtraction are not identical. We recently proved this in an experiment demonstrating that alternated sequences of the creation and annihilation operators give different results when applied to the same initial thermal state[6] (see Figure 3).

In conclusion, photon subtraction and addition are promising tools for engineering quantum light states and proving the principles of quantum physics. In particular, the ability to generate and manipulate a photonic state at will provides an important stepping stone toward applying quantum physics to information and communication technologies.

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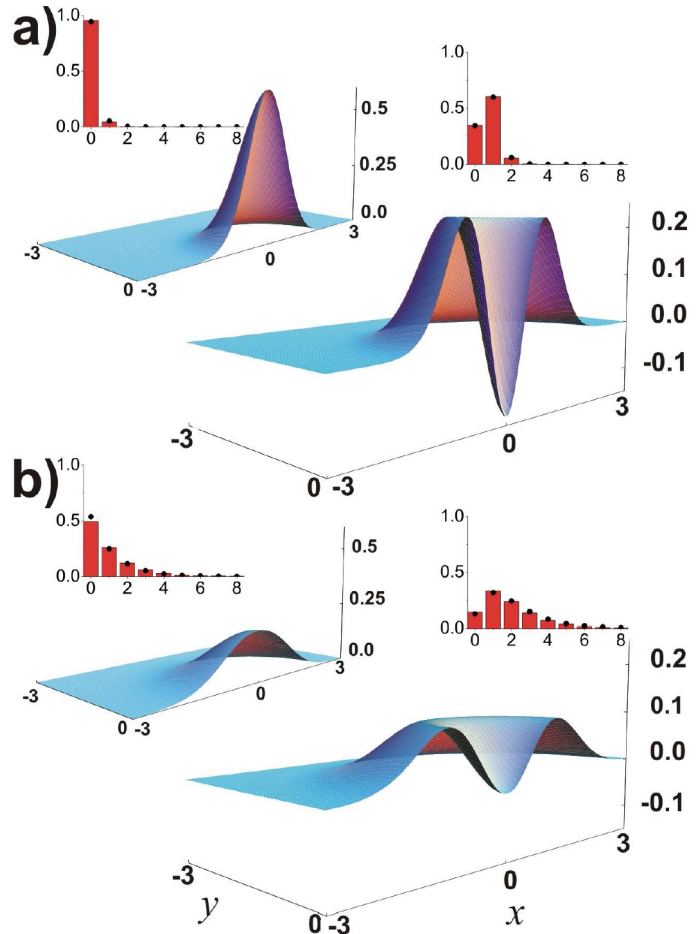
Figure 2. The Wigner functions and photon number probability distributions for thermal fields (left panels) of mean photon number 0.08 (a) and 1.15 (b) and corresponding photon-added thermal fields (right panels).
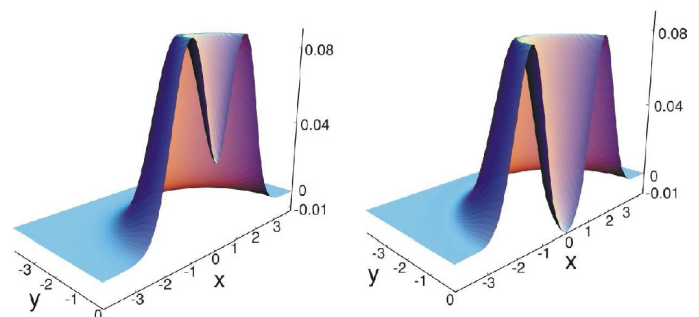

Figure 3. The Wigner functions for the initial thermal field after a photon is first added then subtracted (left), and a photon is first subtracted then added (right).